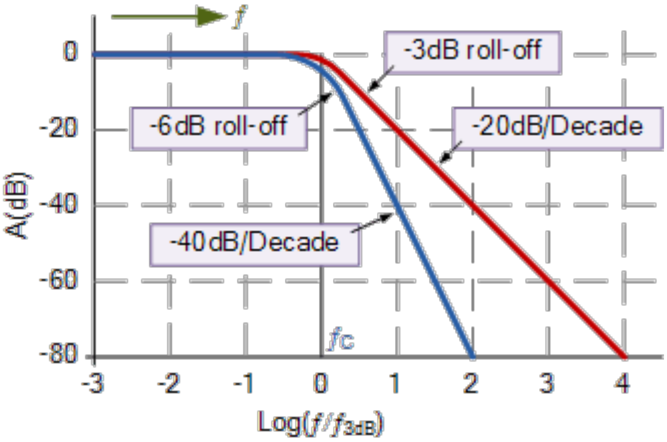


Second Order Filters

Second Order (or two-pole) Filters consist of two RC filter sections connected together to provide a -40dB/decade roll-off rate

Second Order Filters which are also referred to as VCVS filters, because the op-amp is used as a Voltage Controlled Voltage Source amplifier, are another important type of active filter design because along with the active first order RC filters we looked at previously, higher order filter circuits can be designed using them.

In this analogue filters section tutorials we have looked at both passive and active filter designs and have seen that first order filters can be easily converted into second order filters simply by using an additional RC network within the input or feedback path. Then we can define second order filters as simply being: “two 1st-order filters cascaded together with amplification”.

Most designs of second order filters are generally named after their inventor with the most common filter types being: *Butterworth*, *Chebyshev*, *Bessel* and *Sallen-Key*. All these types of filter designs are available as either: low pass filter, high pass filter, band pass filter and band stop (notch) filter configurations, and being second order filters, all have a 40-dB-per-decade roll-off.

The Sallen-Key filter design is one of the most widely known and popular 2nd order filter designs, requiring only a single operational amplifier for the gain control and four passive RC components to accomplish the tuning.

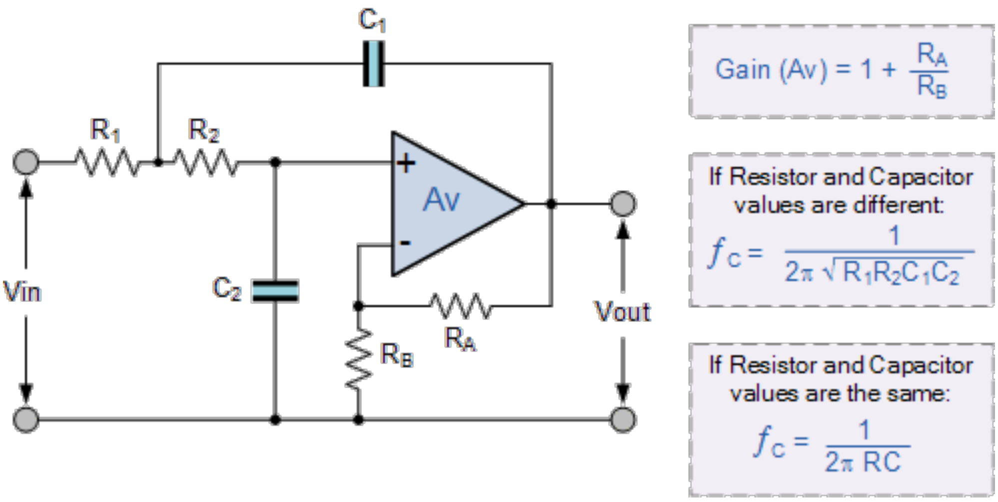
Most active filters consist of only op-amps, resistors, and capacitors with the cut-off point being achieved by the use of feedback eliminating the need for inductors as used in passive 1st-order filter circuits.

Second order (two-pole) active filters whether low pass or high pass, are important in Electronics because we can use them to design much higher order filters with very steep roll-off’s and by cascading together first and second order filters, analogue filters with an n^{th} order value, either odd or even can be constructed up to any value, within reason.

Second Order Low Pass Filter

Second order low pass filters are easy to design and are used extensively in many applic
The basic configuration for a Sallen-Key second order (two-pole) low pass filter is given

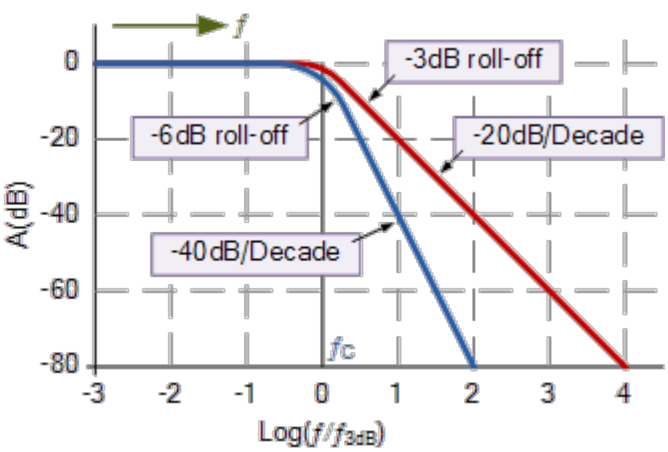
Second Order Low Pass Filter



This second order low pass filter circuit has two RC networks, R1 – C1 and R2 – C2 which give the filter its frequency response properties. The filter design is based around a non-inverting op-amp configuration so the filters gain, A will always be greater than 1. Also the op-amp has a high input impedance which means that it can be easily cascaded with other active filter circuits to give more complex filter designs.

The normalised frequency response of the second order low pass filter is fixed by the RC network and is generally identical to that of the first order type. The main difference between a 1st and 2nd order low pass filter is that the stop band roll-off will be twice the 1st order filters at 40dB/decade (12dB/octave) as the operating frequency increases above the cut-off frequency ƒc, point as shown.

Normalised Low Pass Frequency Response



The frequency response bode plot above, is basically the same as that for a 1st-order filter. The difference this time is the steepness of the roll-off which is -40dB/decade in the stop band. However, second order filters can exhibit a variety of responses depending upon the circuits voltage magnification factor, Q at the the cut-off frequency point.

In active second order filters, the damping factor, ζ (zeta), which is the inverse of Q is normally used. Both Q and ζ are independently determined by the gain of the amplifier, A so as Q decreases the damping factor increases. In simple terms, a low pass filter will always be low pass in its nature but can exhibit a resonant peak in the vicinity of the cut-off frequency, that is the gain can increases rapidly due to resonance effects of the amplifiers gain.

Then Q, the quality factor, represents the "peakiness" of this resonance peak, that is its height and narrowness around the cut-off frequency point, f_C. But a filters gain also determines the amount of its feedback and therefore has a significant effect on the frequency response of the filter.

Generally to maintain stability, an active filters gain must not be more than 3 and is best expressed as:

The Quality Factor, "Q"

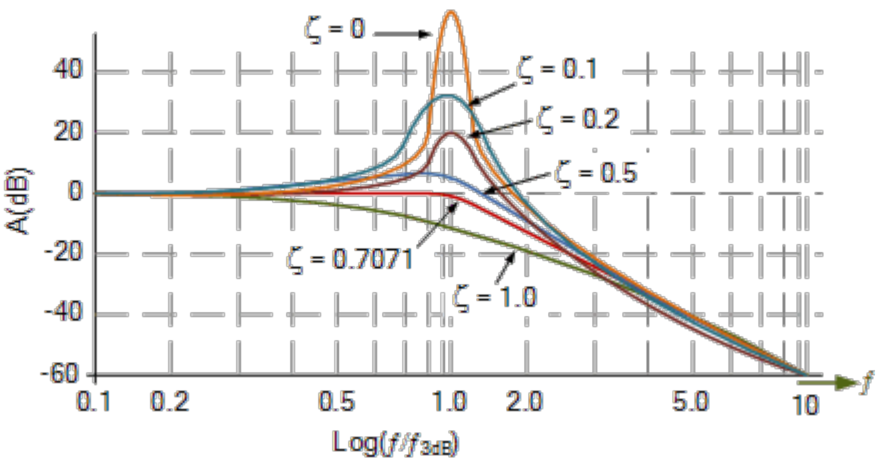
$$A = 3 - (2 \times \zeta)$$

$$\text{Where:} \quad \zeta = \frac{3 - A}{2} = \frac{1}{2Q}$$

$$\therefore A = 3 - \frac{1}{Q}$$

Then we can see that the filters gain, A for a non-inverting amplifier configuration must lie somewhere between 1 and 3 (the damping factor, ζ between zero an 2). Therefore, higher values of Q, or lower values of ζ gives a greater peak to the response and a faster initial roll-off rate as shown.

Second Order Filter Amplitude Response



The amplitude response of the second order low pass filter varies for different values of damping factor, ζ. When ζ = 1.0 or more (2 is the maximum) the filter becomes what is called "overdamped" with the frequency response showing a long flat curve. When ζ = 0, the filters output peaks sharply at the cut-off point resembling a sharp point at which the filter is said to be "underdamped".

Then somewhere in between, ζ = 0 and ζ = 2.0, there must be a point where the frequency response is of the correct value, and there is. This is when the filter is "critically damped" and occurs when ζ = 0.7071.

One final note, when the amount of feedback reaches 4 or more, the filter begins to oscillate by itself at the cut-off frequency point due to the resonance effects, changing the filter into an oscillator. This effect is called self oscillation. Then for a low pass second order filter, both Q and ζ play a critical role.

We can see from the normalised frequency response curves above for a 1st order filter (red line) that the pass band gain stays flat and level (called maximally flat) until the frequency response reaches the cut-off frequency point when: ƒ = ƒr and the gain of the filter reduces past its corner frequency at 1/√2, or 0.7071 of its maximum value. This point is generally referred to as the filters -3dB point and for a first order low pass filter the damping factor will be equal to one, (ζ = 1).

However, this -3dB cut-off point will be at a different frequency position for second order filters because of the steeper -40dB/decade roll-off rate (blue line). In other words, the corner frequency, ƒr changes position as the order of the filter increases. Then to bring the second order filters -3dB point back to the same position as the 1st order filter's, we need to add a small amount of gain to the filter.

So for a Butterworth second order low pass filter design the amount of gain would be: **1.586**, for a Bessel second order filter design: **1.268**, and for a Chebyshev low pass design: **1.234**.

Second Order Filters Example No1

A **Second Order Low Pass Filter** is to be design around a non-inverting op-amp with equal resistor and capacitor values in its cut-off frequency determining circuit. If the filters characteristics are given as: **Q = 5**, and **ƒc = 159Hz**, design a suitable low pass filter and draw its frequency response.

Characteristics given: R1 = R2, C1 = C2, Q = 5 and ƒc = 159Hz

From the circuit above we know that for equal resistances and capacitances, the cut-off frequency point, ƒc is given as:

$$f_C = \frac{1}{2\pi RC}$$

Choosing a suitable value of say, 10kΩ's for the resistors, the resulting capacitor value is calculated as:

$$f_C = \frac{1}{2\pi RC} \qquad \therefore C = \frac{1}{2\pi R f_C}$$

$$C = \frac{1}{2\pi R f_C} = \frac{1}{2\pi \times 10k\Omega \times 159Hz}$$

$$\therefore C = 100nF \text{ or } 0.1\mu F$$

Then for a cut-off corner frequency of **159Hz**, R = 10kΩ and C = 0.1uF.

with a value of **Q = 5**, the filters gain, A is calculated as:

$$Q = 5, \quad \text{and} \quad A = 3 - \frac{1}{Q}$$

$$\therefore A = 3 - \frac{1}{5} = 3 - 0.2 = 2.8$$

We know from above that the gain of a non-inverting op-amp is given as:

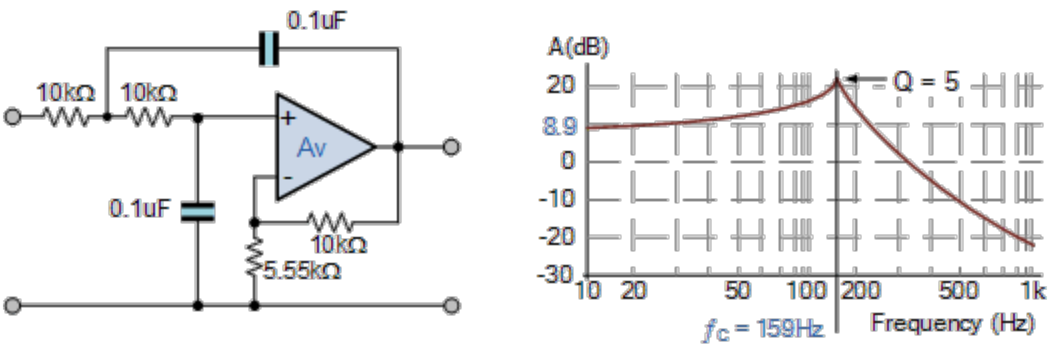
$$A = \frac{Vout}{Vin} = \left(1 + \frac{R_A}{R_B}\right) = 2.8$$

$$\text{Hence:} \quad \frac{R_A}{R_B} = 1.8$$

$$\text{If } R_A = 10k\Omega, \quad \text{then } R_B = 5.55k\Omega$$

Therefore the final circuit for the second order low pass filter is given as:

Low Pass Second Order Filters Circuit



We can see that the peakiness of the frequency response curve is quite sharp at the cut-off frequency due to the high quality factor value, Q = 5. At this point the gain of the filter is given as: Q × A = 14, or about **+23dB**, a big difference from the calculated value of 2.8, (+8.9dB).

But many books, like the one on the right, tell us that the gain of the filter at the normalised cut-off frequency point, etc, etc, should be at the -3dB point. By lowering the value of Q significantly down to a value of **0.7071**, results in a gain of, A = 1.586 and a frequency response which is maximally flat in the passband having an attenuation of -3dB at the cut-off point the same as for a second order butterworth filter response.

So far we have seen that **second order filters** can have their cut-off frequency point set at any desired value but can be varied away from this desired value by the damping factor, ζ. Active filter designs enable the order of the filter to range up to any value, within reason, by cascading together filter sections.

In practice when designing n^{th}-order low pass filters it is desirable to set the cut-off frequency for the first-order section (if the order of the filter is odd), and set the damping factor and corresponding gain for each of the second order sections so that the correct overall response is obtained. To make the design of low pass filters easier to achieve, values of ζ, Q and A are available in tabulated form for active filters as we will see in the Butterworth Filter tutorial. Let's look at another example.

Second Order Filters Example No2

Design a non-inverting second order Low Pass filter which will have the following filter characteristics: Q = 1, and fc = 79.5Hz.

From above, the corner frequency, ƒc of the filter is given as:

$$f_C = \frac{1}{2\pi RC}$$

Choosing a suitable value of 1kΩ for the filters resistors, then the resulting capacitor values are calculated as:

$$f_C = \frac{1}{2\pi RC} \qquad \therefore\ C = \frac{1}{2\pi R f_C}$$

$$C = \frac{1}{2\pi R f_C} = \frac{1}{2\pi \times 1k\Omega \times 79.5Hz}$$

$$\therefore\ C = 2.0\mu F$$

Therefore, for a corner frequency of **79.5Hz**, or 500 rads/s, R = 1kΩ and C = 2.0uF.

With a value of **Q = 1** given, the filters gain A is calculated as follows:

$$Q = \frac{1}{2\xi}, \qquad \therefore\ \xi = \frac{1}{2Q} = \frac{1}{2\times 1} = 0.5$$

$$\xi = 0.5 = \frac{3-A}{2}, \qquad \therefore\ A = 3-2\xi = 2$$

The voltage gain for a non-inverting op-amp circuit was given previously as:

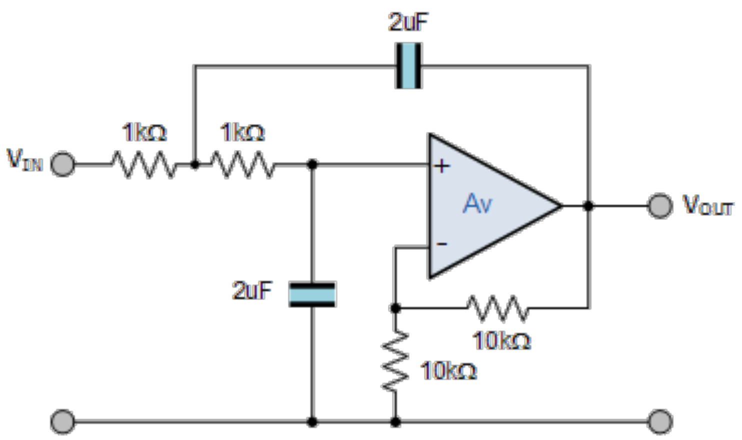
$$A = \frac{Vout}{Vin} = \left(1 + \frac{R_A}{R_B}\right) = 2$$

$$\text{Hence:} \quad \frac{R_A}{R_B} = 1$$

$$\text{If } R_A = 10k\Omega, \text{ then } R_B = 10k\Omega$$

Therefore the second order low pass filter circuit which has a Q of 1, and a corner frequency of 79.5Hz is given as:

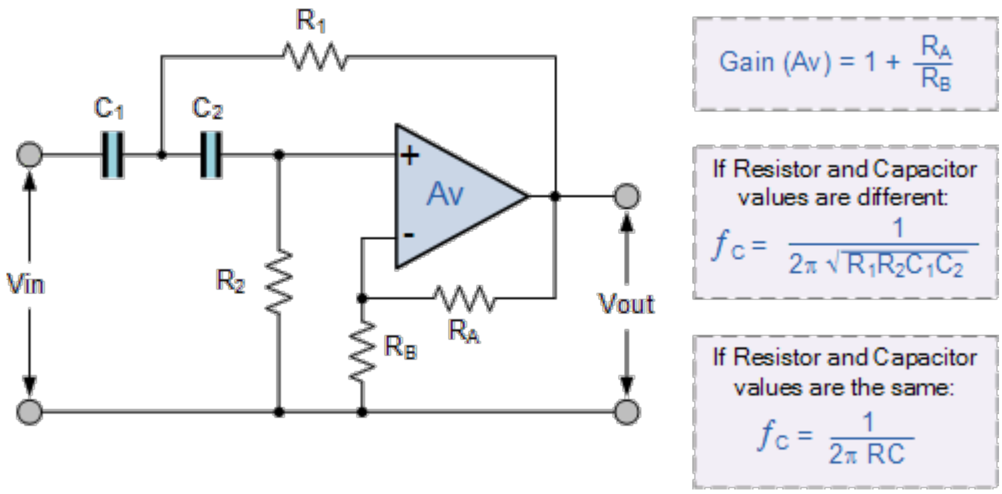
Low Pass Filter Circuit



High Pass Second Order Filters

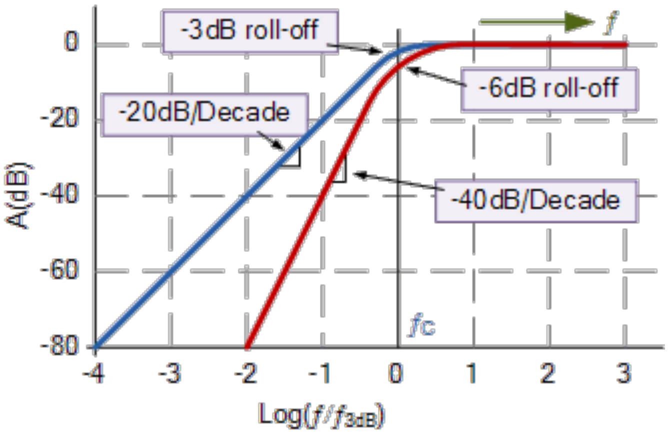
There is very little difference between the second order low pass filter configuration and the second order high pass filter configuration, the only thing that has changed is the position of the resistors and capacitors as shown.

High Pass Second Order Filters



Since second order high pass and low pass filters are the same circuits except that the positions of the resistors and capacitors are interchanged, the design and frequency scaling procedures for the high pass filter are exactly the same as those for the previous low pass filter. Then the bode plot for a 2nd order high pass filter is therefore given as:

Normalised High Pass Frequency Response



As with the previous low pass filter, the steepness of the roll-off in the stop band is -40dB/decade.

In the above two circuits, the value of the op-amp voltage gain, (Av) is set by the amplifiers feedback network. This only sets the gain for frequencies well within the pass band of the filter. We can choose to amplify the output and set this gain value by whatever amount is suitable for our purpose and define this gain as a constant, K.

2nd order Sallen-Key filters are also referred to as positive feedback filters since the output feeds back into the positive terminal of the op-amp. This type of active filter design is popular because it requires only a single op-amp, thus making it relatively inexpensive.

Read more Tutorials inFilters

- 1. Capacitive Reactance
- 2. Passive Low Pass Filter
- 3. Passive High Pass Filter
- 4. Passive Band Pass Filter
- 5. Active Low Pass Filter
- 6. Active High Pass Filter
- 7. Active Band Pass Filter
- 8. Butterworth Filter Design
- 9. Second Order Filters
- 10. State Variable Filter
- 11. Band Stop Filter
- 12. Sallen and Key Filter
- 13. Decibels

51 Comments

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- *Arun Kumar*

  Only low pass filters in non inverting configuration have been shown, Could you also show an example with an inverting opamp configuration.

  Posted on July 12th 2024 | 2:50 am
  Reply
- *A. kahwaji*

  please explain in detail the calculation to arrive at the Butterworth coefficient À=1.586

  Posted on April 30th 2024 | 3:12 pm
  Reply
  - *Christian Thomas*

    Gain = 3 -d, where d = damping = 1/Q

    The Q of a Butterworth 2nd Order section is always 0.707 which is 1/sqrt(2). 3 – 1.414 gives you your 1.586.

    Incidentally, there are tables of what the coefficients are for various filter alignments up to usually 6th or 10th Order. Lancaster's filter cookbook has them, as does Walt Jung's Op Amp handbook (Chapter 7, IIRC).

    Posted on April 30th 2024 | 10:44 pm
    Reply
    - *A kahwaji*

      Mr Thomas
      Thank you

      Posted on May 01st 2024 | 9:54 am
      Reply
- *Henrique*

  Life Saviour.

  Posted on April 17th 2024 | 1:07 pm
  Reply
- *Varun*

  can you please make 2nd order narrow band pass filter having start band frequency of 3khz and stopband frequency of 4khz and of good sensitivity and selectivity

Posted on [April 16th 2024 | 4:10 pm]
[Reply]

- *Harry*

  How do you derive the phase shift of a 2nd order LP filter.

  Posted on [July 20th 2023 | 10:53 am]
  [Reply]

- *Keith*

  I fully agree with the maths.
  but as I said simulations and then testing the non inverting circuit DOES NOT agree with the maths. Thats what I am questioning not the maths

  Posted on [September 18th 2021 | 8:32 am]
  [Reply]

- *Pinky*

  Can I have the derived expression for -40dB per decade second order High pass filter?

  Posted on [May 06th 2021 | 11:11 am]
  [Reply]

- *Very useful*

  Clearly explained with suitable example

  Posted on [April 26th 2021 | 4:28 pm]
  [Reply]

- *asghar*

  Show the design of second-order VCVS 1-dB low pass Chebyshev filter with a gain of 10 and a cutoff frequency of 100 Hz. Use capacitance of 0.1μF. (

  Posted on [January 12th 2021 | 12:15 pm]
  [Reply]

- *Kenneth Thomas*

  I have a unit with a 2nd order sallen-key lpf. In addition to the standard design it has a resistor in parallel with C2. What is the purpose of this resistor and how is it used in performance calculation.

  Posted on [November 09th 2020 | 1:38 pm]
  [Reply]

- *Darshan Parmar*

  How we can do is:-
  Design a second order low pass filter using cascading of two first order filters for cut-off frequency 2 KHz. Compare its output with second order filter designed for same cut-off 2KHz in terms of gain and roll off in stop band. Give your comments by comparing both options.

  Posted on [April 06th 2020 | 10:45 am]
  [Reply]

- *raj panchal*

  it is very helpful on the night before exam .
  i am so thankful to you because of you only i passed my university exam.

  Posted on [November 29th 2019 | 3:12 pm]

Reply

- *Shahidul Islam Mahmud*

  i can not clearly understand about second order low pass & high pass filter calculation perfectly.

  Posted on November 10th 2019 | 2:04 pm
  Reply
- *Delroy Frater*

  There needs to be example problems and solutions for the second order high pass filter as is the case for second order low pass filter.

  Posted on March 25th 2019 | 7:40 am
  Reply
- *PRADYUMNA KUMAR DAS*

  It is give to design 2nd order lpf at a higher cutoff freq. Of 5khz. So what is the gain if we dont know the quality factor??

  Posted on March 04th 2019 | 6:59 am
  Reply
- *Rocko*

  Thanks for the great article. Just want to make sure I got things right.

  A 2nd order filter can be designed as a 'passive' or as an 'active (with opAmp, etc.)' implementation. According to the above, the Q factor is related to the gain factor around the cut-off point. In the passive design, this gain factor is not controllable.

  If so, is it true to say that a passive design has a 'given' Q factor which can not be controlled by the designer or the user of the filter?

  Posted on November 18th 2018 | 1:22 pm
  Reply
- *Abhilash*

  it's good to read…. all enough matter is there to study

  Posted on November 11th 2018 | 6:39 pm
  Reply
- *Inna Petrova Nikolova*

  I don't like the stupidity in this article

  Posted on November 02nd 2018 | 9:39 am
  Reply
- *Art*

  Thank you very much for the detailed and informative tutorial. But want to make sure I have something right, after reading the statement:

  "Most designs of second order filters are generally named after their inventor with the most common filter types being: Butterworth, Chebyshev, Bessel and Sallen-Key. "

  I had thought that Sallen-Key referred to a filter circuit topology, not a filter characteristic (denominator polynomial class). e.g., Q for a Second Order SK circuit can be set to have the characteristics of a Butterworth (Q=.707), Bessel (Q=.557) or Chebychev (Q=1.55) filter. (For second order filters, the only distinction between these filter characteristics seems to be the value of Q.) Or does SK actually imply both a topology AND and filter characteristic? Thanks.

Posted on October 24th 2018 | 6:23 am
Reply

- More

- *Michael Abiyeye*

  Can you pls domostrate how oscilloscopecan be used to perform these filters?

  Posted on May 20th 2018 | 9:00 pm
  Reply